

## Problem Sheet 7

Deadline: **Monday 28 November, 5:00.**

Hand in to **the drop box** in the undergraduate common room (maths department, room 502).

**Hand in the questions marked with an asterisk (\*).**

One mark will be deducted if you do not **staple your work**.

1) Use separation of variables to find the general solutions of the following:

a)  $\frac{dy}{dx} = \frac{1}{x^2}$

\* d)  $\frac{dy}{dx} = 2xy$

b)  $\frac{dy}{dx} = \frac{y}{x}$

e)  $xy \frac{dy}{dx} = \frac{1+x^2}{1+y}$

\* c)  $2x \cos y = (x^2 - 3) \sec y \frac{dy}{dx}$

\* f)  $\frac{dx}{dy} + \frac{1}{x^2} = 0$  [Notice:  $\frac{dx}{dy}$ , not  $\frac{dy}{dx}$ ]

2) Use integrating factors to find the general solutions of the following:

a)  $\frac{dy}{dx} + 2y = 1$

\* d)  $\frac{dy}{dx} = \cos x - y \tan x$   
[Hint: to integrate  $\tan x$ , use  $u = \sec x$ ]

\* b)  $\frac{dy}{dx} + 2xy = 2x$

\* e)  $x^2 \frac{dy}{dx} + xy + 1 = 0$

c)  $\frac{dy}{dx} + 2y = e^x$

f)  $e^x \frac{dy}{dx} = 1 - ye^x$

3) Solve the following initial value problems:

a)  $\frac{dy}{dx} + 2y = 1, \quad y(0) = 0$

\* d)  $x \frac{dy}{dx} - 3y = 0, \quad y(1) = 5$

b)  $\frac{dy}{dx} = \frac{1}{x^2}, \quad y(1) = 8$

\* e)  $e^{-x} \frac{dy}{dx} + y = 1, \quad y(0) = 1$

\* c)  $\frac{dy}{dx} + \frac{y}{x} = 6, \quad y(1) = 4$

\* f)  $\frac{dy}{dx} = (xy)^2, \quad y(1) = 1$