MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 5 Solutions

1) Find the following:

* b) $\int 4(x+\sin x)^3(1+\cos x)\ dx$ [1 mark]

$$\begin{aligned}
u &= x+\sin x\\
\frac{du}{dx} &= 1+\cos x\\
du &= (1+\cos x)\ dx\\
\int 4(x+\sin x)^3(1+\cos x)\ dx &= \int 4u^3\ du\\
&= u^4+c\\
&= (x+\sin x)^4+c
\end{aligned}$$

* d) $\int e^{x+e^x}\ dx$ [1 mark]

$$\begin{aligned}
u &= e^x\\
\frac{du}{dx} &= e^x\\
du &= e^x\ dx\\
\int e^{x+e^x}\ dx &= \int e^{e^x}e^x\ dx\\
&= \int e^u\ du\\
&= e^u+c\\
&= e^{e^x}+c
\end{aligned}$$

* g) $\int (10x-8)^5\ dx$ [1 mark]

$$\begin{aligned}
u &= 10x-8\\
du &= 10\ dx\\
\int (10x-8)^5\ dx &= \frac{1}{10}\int u^5\ du\\
&= \frac{1}{60}u^6+c\\
&= \frac{1}{60}(10x-8)^6+c
\end{aligned}$$

* h) $\displaystyle\int x\sin(x^2+5)\ dx$ [1 mark]

$$\begin{aligned} u &= x^2+5 \\ du &= 2x\ dx \\ \int x\sin(x^2+5)\ dx &= \frac{1}{2}\int \sin u\ du \\ &= -\frac{1}{2}\cos u + c \\ &= -\frac{1}{2}\cos(x^2+5) + c \end{aligned}$$

* j) $\displaystyle\int \frac{\sec^2 x}{\tan x}\ dx$ [1 mark]

$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x\ dx \\ \int \frac{\sec^2 x}{\tan x}\ dx &= \int \frac{1}{u}\ du \\ &= \ln u + c \\ &= \ln(\tan x) + c \end{aligned}$$

* k) $\displaystyle\int \frac{x^5+x^2-1}{x^6+2x^3-6x}\ dx$ [1 mark]

$$\begin{aligned} u &= x^6+2x^3-6x \\ du &= 6x \qquad\qquad 5+6x^2-6 \\ \int \frac{x^5+x^2-1}{x^6+2x^3-6x}\ dx &= \frac{1}{6}\int \frac{1}{u}\ du \\ &= \frac{1}{6}\ln u + c \\ &= \frac{1}{6}\ln(x^6+2x^3-6x) + c \end{aligned}$$

* m) $\displaystyle\int \frac{x}{1-x^2} + \cos(4x+1)\ dx$ [2 mark]

$$\begin{aligned} \int \frac{x}{1-x^2} + \cos(4x+1)\ dx &= \int \frac{x}{1-x^2}\ dx + \int \cos(4x+1)\ dx \\ u &= 1-x^2 \qquad\qquad v = 4x+1 \\ du &= -2x\ dx \qquad\qquad dv = 4\ dx \\ &= -\frac{1}{2}\int \frac{1}{v}\ du + \frac{1}{4}\int \cos(v)\ dv \\ &= -\frac{1}{2}\ln v + \frac{1}{4}\sin(v) + c \\ &= -\frac{1}{2}\ln(1-x^2) + \frac{1}{4}\sin(4x+1) + c \end{aligned}$$

* n) $\displaystyle\int xe^{x^2} - \frac{e^{2x} - e^{-2x}}{(e^x + e^{-x})^2}\ dx$ [2 mark]

$$\int xe^{x^2} - \frac{e^{2x} - e^{-2x}}{(e^x + e^{-x})^2}\ dx = \int xe^{x^2}\ dx - \int \frac{e^{2x} - e^{-2x}}{(e^x + e^{-x})^2}\ dx$$

$$u = x^2 \qquad\qquad v = (e^x + e^{-x})^2$$

$$du = 2x\ dx \qquad\qquad dv = 2(e^x + e^{-x})(e^x - e^{-x})\ dx$$

$$= 2(e^{2x} + e^{-2x})\ dx$$

$$\begin{aligned}
&= \frac{1}{2}\int e^u\ du - \frac{1}{2}\int \frac{1}{v}\ dv \\
&= \frac{1}{2}e^u - \frac{1}{2}\ln v + c \\
&= \frac{1}{2}e^{x^2} - \frac{1}{2}\ln\left((e^x + e^{-x})^2\right) + c \\
&= \frac{1}{2}e^{x^2} - \ln(e^x + e^{-x}) + c
\end{aligned}$$