1.4.4 Complex numbers

To allow ourselves to solve all quadratics, not just those with real roots, we introduce i:

Definition: i

$$\mathrm{i} = \sqrt{-1}$$

Numbers of the form $b\mathrm{i}$ are called imaginary numbers. Numbers of the form $a + b\mathrm{i}$ are called complex numbers. They can be used as follows:

Example
To solve $x^2 + 4 = 0$:

$$\begin{aligned} x^2 + 4 &= 0 \\ x^2 &= -4 \\ x &= \pm\sqrt{-4} \\ x &= 2\mathrm{i} \text{ or } -2\mathrm{i} \end{aligned}$$

To solve $x^2 + 6x + 13 = 0$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{36 - 4 \times 13}}{2} \\ &= \frac{-6 \pm \sqrt{-16}}{2} \\ &= \frac{-6 \pm 4\mathrm{i}}{2} \\ &= -3 + 2\mathrm{i} \text{ or } -3 - 2\mathrm{i} \end{aligned}$$

To add, multiply and subtract complex numbers, treat i like a variable and remember that $\mathrm{i}^2 = -1$:

Example

$$(2 + 4\mathrm{i}) + (3 - 2\mathrm{i}) = 5 + 2\mathrm{i}$$

$$\begin{aligned} (2 + 4\mathrm{i}) \times (3 - 2\mathrm{i}) &= 6 + 12\mathrm{i} - 4\mathrm{i} - 8\mathrm{i}^2 \\ &= 6 + 12\mathrm{i} - 4\mathrm{i} + 8 \\ &= 14 + 8\mathrm{i} \end{aligned}$$