MATH6103 Differential & Integral Calculus

Progress Checking Test Solutions

1) Find $\dfrac{dy}{dx}$ when $y = e^{\sin x}$

By the chain rule:

$$\begin{aligned}\frac{d}{dx}\left[e^{\sin x}\right] &= e^{\sin x} \cdot \frac{d}{dx}[\sin x] \\ &= e^{\sin x} \cdot \cos x\end{aligned}$$

2) Find $\dfrac{dy}{dx}$ when $y = e^x \sin x$

By the product rule:

$$\begin{aligned}\frac{d}{dx}[e^x \sin x] &= e^x \frac{d}{dx}[\sin x] + \sin x \frac{d}{dx}[e^x] \\ &= e^x \cos x + e^x \sin x\end{aligned}$$

3) Find $\dfrac{dy}{dx}$ when $y = \sin(e^x)$

By the chain rule:

$$\begin{aligned}\frac{d}{dx}[\sin(e^x)] &= \cos(e^x) \frac{d}{dx}[e^x] \\ &= e^x \cos(e^x)\end{aligned}$$

A satellite is orbiting the moon. Its position can be described in polar co-ordinates by the equation

$$r = \frac{300}{2+\cos\theta},$$

where r is the distance from the moon (in km) and θ is the angle (in radians).

4a) Find $\dfrac{dr}{d\theta}$.

$$\begin{aligned} r &= \frac{300}{2+\cos\theta} \\ &= 300(2+\cos\theta)^{-1} \end{aligned}$$

By the chain rule:

$$\begin{aligned} \frac{d}{dx}\left[300(2+\cos\theta)^{-1}\right] &= 300\frac{d}{dx}\left[(2+\cos\theta)^{-1}\right] \\ &= 300 \cdot -(2+\cos\theta)^{-2}\frac{d}{dx}\left[2+\cos\theta\right] \\ &= 300 \cdot -(2+\cos\theta)^{-2} \cdot -\sin\theta \\ &= \frac{300\sin\theta}{(2+\cos\theta)^2} \end{aligned}$$

[The could instead have been done by the quotient rule.]

4b) Solve $\dfrac{dr}{d\theta} = 0$.

The equation

$$\frac{300\sin\theta}{(2+\cos\theta)^2} = 0$$

is zero when $\sin\theta = 0$. $\sin\theta = 0$ when:

$$\theta = 0, \pi, 2\pi, 3\pi, ...$$

4c) Find the minimum distance from the moon which the satellite reaches during its orbit.

During one orbit, θ goes from 0 to 2π. In this range, the minimum must be at either $\theta = 0$ or $\theta = \pi$.
When $\theta = 0$, $r = 100km$. When $\theta = \pi$, $r = 300km$. Therfore the minimum distance is $100km$.