

**MATH6103 Differential & Integral Calculus**  
**MATH6500 Elementary Mathematics for Engineers**

**Problem Sheet 3 Mark Scheme**

1) Differentiate the following functions:

\* a)  $a(x) = x^2 - \frac{1}{x^2}$  [1 mark]

$$a'(x) = 2x + 2x^{-3}$$

\* d)  $d(x) = 359x^{17}$  [1 mark]

$$d'(x) = 6103x^{16}$$

\* f)  $f(x) = \sin(x^2)$  [1 mark]

$$f'(x) = 2x \cos(x^2)$$

\* g)  $g(x) = xe^x$  [1 mark]

$$g'(x) = e^x + xe^x$$

2) Differentiate the following functions:

\* a)  $i(x) = \cos(4 + 3x^2)$  [1 mark]

$$i'(x) = -6x \sin(4 + 3x^2)$$

\* d)  $l(x) = \cos(\sin x)$  [1 mark]

$$l'(x) = -\sin(\sin x) \cos x$$

\* g)  $o(x) = \sqrt{\sin x + \cos x}$  [1 mark]

$$\begin{aligned} o'(x) &= \frac{1}{2}(\sin x + \cos x)^{-\frac{1}{2}}(\cos x - \sin x) \\ &= \frac{\cos x - \sin x}{2\sqrt{\sin x + \cos x}} \end{aligned}$$

3) Find the  $x$  co-ordinates of the turning points of the following:

\* b)  $s(x) = x^3 - 108x$  [1 mark]

$$\begin{aligned} s'(x) &= 3x^2 - 108 \\ 3x^2 - 108 &= 0 \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

\* c)  $r(x) = x^3 + 3x^2 + 2x - 8$  [1 mark]

$$r'(x) = 3x^2 + 6x + 2$$

$$3x^2 + 6x + 2 = 0$$

$$x = \frac{-6 \pm \sqrt{12}}{6}$$

$$x = \frac{-3 \pm \sqrt{3}}{3}$$

\* d)  $t(x) = \sin x + \cos x$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$  [1 mark]

$$t'(x) = \cos x - \sin x$$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$1 = \tan x$$

$$x = \frac{\pi}{4}$$