3.5 Applications of integration

There are many real life situations in which quantities are derivatives or integrals of each other. Some are given in the table below. Differentiating moves to the right in the table (following the arrows). Integrating moves to the left (against the arrows).

Distance	$\to$	Speed	$\to$	Acceleration
Energy	$\to$	Force		
Prices	$\to$	Inflation	$\to$	Rate of change of inflation
Debt	$\to$	Deficit	$\to$	Increase/reduction in deficit
Bank balance	$\to$	Interest		
Population size	$\to$	Birth and death rates		
Something	$\to$	Rate of change of that thing		

All of the above would involve $\frac{d}{dt}$ as they are the rate of change of a quantity over **time**.

3.5.1 Finding a distance by the integral of velocity

If you know the velocity $v(t)$, then the distance s as a function of time, i.e. $s = s(t)$, is

$$s(t) = \int v(t)\,dt, \quad (\text{since } s'(t) = v(t)).$$

Similarly, if you know the acceleration $a(t)$, then the velocity can be found by the integral of $a(t)$,

$$v(t) = \int a(t)\,dt \quad (\text{since } v'(t) = a(t))$$

Example
A ball is thrown down from a tall building with an initial velocity of 100ft/sec. Then its velocity after t seconds is given by $v(t) = 32t + 100$. How far does the ball fall between 1 and 3 seconds of elapsed time?
First let us write the distance as the integral of the velocity, that is

$$s(t) = \int v(t) \;=\; \int 32t + 100\,dt = 16t^2 + 100t + C.$$

Then the distance fallen is given by

$$s(3) - s(1) = (16t^2 + 100t + C)\big|_{t=3} - (16t^2 + 100t + C)\big|_{t=1} = 328 \text{ ft}.$$

Notice that

$$\begin{aligned}(16t^2 + 100t + C)\big|_{t=3} - (16t^2 + 100t + C)\big|_{t=1} &= (16t^2 + 100t + C)\big|_1^3 \\ &= \int_1^3 (32t + 100)\,dt \\ &= \int_1^3 v(t)\,dt.\end{aligned}$$

3.5.2 Finding the area between two curves

Example
To find the area between the curves $y = x^2$ and $y = 50 - x^2$...