3.4 Some difficulties

Example
Consider the integral of $1/x^2$ on the interval $[-1, 1]$.

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_{-1}^{1} = \left[-\frac{1}{1}\right] - \left[-\frac{1}{-1}\right] = -2.$$

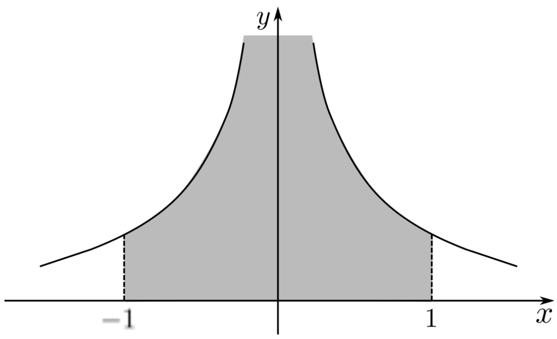

Figure 3.4: Integrating to find the shaded area under the curve $y = \frac{1}{x^2}$ on the interval $[-1, 1]$?

However, the area under the curve in this interval is not -2! What is wrong here?

In the above example, the integral was not the area under the curve because there was an asymptote at $x = 0$. We must be sure that the function is defined over the whole domain before we integrate.

Example
Consider the function $f(x) = x^3$. Then the integral over the interval $[-1, 1]$ is

$$\int_{-1}^{1} x^3\,dx = \left[\frac{1}{4}x^4\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0.$$

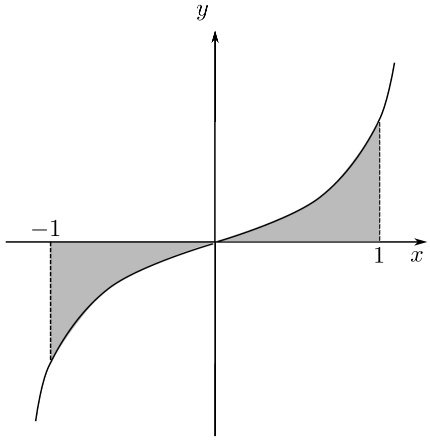

Figure 3.5: The area under the curve $y = x^3$ on the interval $[-1, 1]$.

In this case, the areas cancel out as area below the x-axis is negative. The shaded area is actually given by

$$\int_0^1 x^3\, dx + \left|\int_{-1}^0 x^3\, dx\right| = \left[\frac{1}{4}x^4\right]_0^1 + \left|\left[\frac{1}{4}x^4\right]_{-1}^0\right| = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$