

Problem Sheet 1 Solutions

1) Solve the following equations:

a) $4x + 16 = 0$

$$\begin{aligned}4x + 16 &= 0 \\4x &= -16 \\x &= -4\end{aligned}$$

b) $x^2 + 5x - 24 = 0$

$$\begin{aligned}x^2 + 5x - 24 &= 0 \\(x - 3)(x + 8) &= 0 \\x &= -8 \text{ or } 3\end{aligned}$$

c) $x^2 - 5x - 24 = 0$

$$\begin{aligned}x^2 - 5x - 24 &= 0 \\(x + 3)(x - 8) &= 0 \\x &= -3 \text{ or } 8\end{aligned}$$

d) $x^2 + 5x = -6$

$$\begin{aligned}x^2 + 5x &= -6 \\x^2 + 5x + 6 &= 0 \\(x + 2)(x + 3) &= 0 \\x &= -3 \text{ or } -2\end{aligned}$$

e) $x^2 + 5x = 0$

$$\begin{aligned}x^2 + 5x &= 0 \\x(x + 5) &= 0 \\x &= -5 \text{ or } 0\end{aligned}$$

f) $3x^2 + 8x + 1 = 0$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-8 \pm \sqrt{8^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} \\&= \frac{-8 \pm \sqrt{64 - 12}}{6} \\&= \frac{-8 \pm \sqrt{52}}{6} \\&= \frac{-8 \pm 2\sqrt{13}}{6} \\&= \frac{-4 \pm \sqrt{13}}{3} \\&= \frac{-4 + \sqrt{13}}{3} \text{ or } \frac{-4 - \sqrt{13}}{3}\end{aligned}$$

2) Solve the following equations:

a) $x^3 + 4x^2 + x - 6 = 0$ (Clue: $x = 1$ is one solution)

$$\begin{aligned}x^3 + 4x^2 + x - 6 &= 0 \\(x - 1)(x^2 + 5x + 6) &= 0 \text{ by long division} \\(x - 1)(x + 2)(x + 3) &= 0 \\x &= -3 \text{ or } -2 \text{ or } 1\end{aligned}$$

b) $x^3 - 5x^2 + 60x - 224 = 0$ (Clue: $x = 4$ is one solution)

$$\begin{aligned}x^3 - 5x^2 + 60x - 224 &= 0 \\(x - 4)(x^2 - x + 56) &= 0 \text{ by long division} \\(x - 1)(x - 7)(x - 8) &= 0 \\x &= 1 \text{ or } 7 \text{ or } 8\end{aligned}$$

c) $x^3 - 2x^2 - 4x = 0$

$$\begin{aligned}
 x^3 - 2x^2 - 4x &= 0 \\
 x(x^2 - 2x - 4) &= 0 \\
 x = 0 \text{ or } x^2 - 2x - 4 &= 0
 \end{aligned}$$

To solve $x^2 - 2x - 4 = 0$, use the formula:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x &= \frac{2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot -4}}{2 \cdot 1} \\
 x &= \frac{2 \pm \sqrt{20}}{2} \\
 x &= \frac{2 \pm 2\sqrt{5}}{2} \\
 x &= 1 \pm \sqrt{5}
 \end{aligned}$$

So overall:

$$x = 0 \text{ or } 1 + \sqrt{5} \text{ or } 1 - \sqrt{5}$$

3) Which of the following are functions?

If they are functions, find their range.

If not, explain why not.

a) $a(x) = x + 2$ with domain \mathbb{R}

It is a function with range \mathbb{R} .

b) $b(x) = \sqrt{x}$ with domain \mathbb{N} .

It is a function with range $\{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$ (a subset of \mathbb{R}).

c) $c(x) = x^2 + 3$ with domain \mathbb{R} .

It is a function with range $\{x \in \mathbb{R} : x \geq 3\}$

d) $d(x) = \sqrt{x}$ with domain \mathbb{R} .

It is not a function as you cannot square root negative numbers.

OR

It is a function with range $\mathbb{R}^+ \cup \{ai : a \in \mathbb{R}\}$